

# A Calculus for Secure Mobility

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## Introduction

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Our goal is to combine:

- **cryptography**
- **mobility**: processes/programs being sent on the network
- **locations**: protections by firewalls for instance

to represent all security aspects of network applications, such as Java applets.

# Overview

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Design choices

The calculus (syntax, part of the semantics)

Examples

Observational equivalence

## Representing cryptography

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Many different variants of the **spi-calculus** [Abadi, Gordon, I&C'99]  
= pi calculus + cryptographic primitives.

We focus on the **applied pi calculus** [Abadi, Fournet, POPL'01]  
which is not limited to a small set of built-in primitives:  
cryptographic primitives can be defined by **equational theories**.

Example: symmetric encryption

$$\text{decrypt}(\text{encrypt}(x, y), y) = x$$

optionally, we can add  $\text{decrypt}(M, N) = \text{wrong}$  when  $M \neq \text{encrypt}(L, N)$   
for all  $L$ .

## Representing cryptography

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A more complex example: Diffie-Hellman key agreements use modular exponentiation and can be modeled using the equational theory of [Meadows, Narendran, WITS'02]

$$x * (y * z) = (x * y) * z$$

$$x * y = y * x$$

$$x * 1 = x$$

$$x * x^{-1} = 1$$

$$\text{exp}(x, 1) = x$$

$$\text{exp}(\text{exp}(x, y), z) = \text{exp}(x, y * z)$$

We can represent data structures, public-key encryption, signatures, hash functions, macs, Diffie-Hellman key agreements, XOR, ...

## Representing mobility

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- Processes should be sent and received **explicitly**, as in current real networks.

Excludes the **ambient calculus** [Cardelli, Gordon, TCS'00] and variants, in which processes move **by themselves**.

Closer to the **seal calculus** [Vitek, Castagna, IPL'99]

- Processes should be manipulated by cryptographic functions, for example, to encrypt them before sending them on the network.

⇒ **processes as data**, higher-order calculus

## Processes as data

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We use special functions:

**pack** converts a process into a piece of data

**exec** converts a piece of data into a process

- A packed process is  $\text{pack}((M_1, \dots, M_n), \lambda x_1, \dots, x_n.P)$  with  $fn(P) = \emptyset$  and  $fv(P) \subseteq \{x_1, \dots, x_n\}$ .
  - $(M_1, \dots, M_n)$  represents the **data** of the packed process.
  - $\lambda x_1, \dots, x_n.P$  represents its **code**.
- When the packed process is executed, it runs  $P\{M_1/x_1, \dots, M_n/x_n\}$ :  
$$\text{exec}(\text{pack}((M_1, \dots, M_n), \lambda x_1, \dots, x_n.P)) \rightarrow P\{M_1/x_1, \dots, M_n/x_n\}$$
- When the packed process is sent in the clear on the network, it reveals its contents, in particular its data  $(M_1, \dots, M_n)$ .

## Processes as data (continued)

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The definition of packed processes ensures **separation of code and data**, which is important for security.

The following processes have the same effect when they are executed, but reveal different pieces of data:

$$\mathbf{pack}((s, k, a), \lambda x, y, z. \bar{z}^\uparrow(\mathbf{encrypt}(x, y)))$$

reveals  $s, k, a$  when it is sent in clear over the network, whereas

$$\mathbf{pack}((\mathbf{encrypt}(s, k), a), \lambda x, z. \bar{z}^\uparrow(x))$$

only reveals  $\mathbf{encrypt}(s, k)$  and  $a$ , so it does not reveal  $s$  when the adversary does not have  $k$ .



## Processes as data (continued)

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- **getdata** allows the adversary to obtain data from a packed process:

$$\text{getdata}(\text{pack}((M_1, \dots, M_n), \lambda \vec{x}. P)) = (M_1, \dots, M_n)$$

- The adversary can also **compare the code of processes** (by building a packed process with the same data and its own code).
- We can easily add a function to obtain the code of a process as a term, which would make it possible to modify received processes.

# Syntax

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Terms represent **messages** (data).

Processes represent **code**.

$M, N ::=$

$x, y, z$

$a, b, c, k$

$f(M_1, \dots, M_n)$

$\text{pack}(\vec{M}, \lambda \vec{x}. P)$

(with  $fn(P) = \emptyset, fv(P) \subseteq \vec{x}$ )

terms

variable

name

function application

packed process

## Syntax

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$\eta ::=$	communication target
$M$	down
$\uparrow$	up
$\star$	local
$P, Q ::=$	processes
$0$	
$P \mid Q$	
$!P$	
$(\nu a)P$	
$M^\eta(\lambda x).P$	input
$\overline{M}^\eta(M').P$	output
$M[P]$	location
$\text{exec}(M)$	execute packed process
$\text{if } M = N \text{ then } P \text{ else } Q$	conditional

## Key part of the semantics: Communications with locations

Semantics: a structural equivalence relation  $\equiv$   
and a reduction relation  $\rightarrow$ .

Locations  $M[P]$  communicate internally and with locations immediately inside or outside them:

- **local:**  $a^*(\lambda y).P' \mid \bar{a}^*(M).Q' \rightarrow P'\{M/y\} \mid Q'$
- **down:**  $c[a^\uparrow(\lambda y).P' \mid R] \mid \bar{a}^c(M).Q' \rightarrow c[P'\{M/y\} \mid R] \mid Q'$
- **up:**  $a^c(\lambda y).P' \mid c[\bar{a}^\uparrow(M).Q' \mid R] \rightarrow P'\{M/y\} \mid c[Q' \mid R]$

Similar to the **boxed ambients** [Bugliesi, Castagna, Crafa, TACS'01]

## Example: firewall

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Locations can be used to model **firewalls**: when the adversary does not have  $c$ , it cannot send messages through  $c[\dots]$ .

- A **perfect firewall** is defined by

$$(\nu c)c[P]$$

Nothing goes through this firewall.

- A **firewall that leaves some channels open** can be represented by

$$(\nu c)(c[P] \mid !a^*(\lambda x).\bar{a}^c(x) \mid !b^c(\lambda x).\bar{b}^*(x))$$

The process forwards messages on channel  $a$  from outside to inside  $c[\dots]$  and on channel  $b$  from inside to outside  $c[\dots]$ .

## Example: applet

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Processes sent as messages can be used to model **applets** (such as Java applets).

Java applets are often signed for security:

$$\begin{array}{l} C \rightarrow S: \text{ "Send me your applet" } \\ S \rightarrow C: \{ \mathbf{pack}(M, \lambda y.P) \}_{sk_S} \end{array}$$

$(\nu sk_S) \text{ let } pk_S = \mathbf{pk}(sk_S) \text{ in } \bar{c}^*(pk_S).$

$((C) \quad \bar{c}^*(\text{SendApplet}).c^*(\lambda x).\text{exec}(\text{checksign}(x, pk_S)))$

$| (S) \quad c^*(\lambda x).\text{if } x = \text{SendApplet} \text{ then } \bar{c}^*(\text{sign}(\mathbf{pack}(M, \lambda y.P), sk_S)))$

## Example: applet with a sandbox

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A well-placed sandbox can **protect against malicious applets**:

$$c^*(\lambda x).(\nu sb)(sb[\mathbf{exec}(x)] \mid P) \mid Q$$

where  $P$  models the **Java standard library**. Its communications with the applet model method calls and returns.

The applet can communicate to the external world only through  $P$ . So  $P$  can restrict these communications for security.

The process  $Q$  is protected from the applet.  
(It is outside of the sandbox.)





## Observational equivalence

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$P$  and  $Q$  are **observationally equivalent**,  
 $P \approx Q$ , when the adversary cannot distinguish  $P$  from  $Q$ .

$\approx$  is the largest symmetric relation  $\mathcal{R}$  between closed processes such that  $P \mathcal{R} Q$  implies:

- if  $P$  emits on channel  $a$ , then  $Q$  emits on  $a$ ;
- if  $P \rightarrow^* P'$  then there exists  $Q'$  such that  $Q \rightarrow^* Q'$  and  $P' \mathcal{R} Q'$ ;
- $C[P] \mathcal{R} C[Q]$  for all closing evaluation contexts  $C[\ ]$ .  
( $C ::= [] \quad (\nu a)C \quad P \mid C \quad C \mid P \quad M[C]$ )

## Secrecy (non-interference)

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Observational equivalence can be used to define various security properties, for example **secrecy**:

Let  $P$  be a process with free variables  $x_1, \dots, x_n$ .

$P$  preserves the secrecy of  $x_1, \dots, x_n$  if and only if for all  $\sigma$  and  $\sigma'$  substitutions of domain  $x_1, \dots, x_n$ ,

$$P\sigma \approx P\sigma'$$

## The adversary can observe the code of processes

The adversary can **observe the code** of packed processes, by testing equality with processes he builds

$$c^n(\lambda x).if\ x = \mathbf{pack}(\mathbf{getdata}(x), \lambda \vec{y}.P)\ \text{then } \bar{a}^*(x)\ \text{else } 0$$

tests equality between the code of  $x$  and  $P$ .

Important consequence: for the equivalence, the language behaves **like a first-order language**.

In contrast, in most higher-order languages, the adversary can observe processes only by running them.

## Observational equivalence as labeled bisimilarity

Observational equivalence proofs are difficult, because we have to prove properties **for all** contexts.

**Labeled bisimilarity** makes the proofs much easier by avoiding this quantification.

## Agents

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**Agents** represent processes as well as the current knowledge of the adversary.

$$A = (\nu \vec{a})(\{M_1/x_1, \dots, M_n/x_n\}, P)$$

The adversary has the terms  $M_1, \dots, M_n$ , but sometimes cannot inspect the whole structure of these terms, because it does not a priori have the names  $\vec{a}$ .

$$(\nu a, b, k)(\{\text{encrypt}((a, b), k)/x\}, P)$$

The adversary cannot see that  $x$  is the encryption of a pair.

The structural equivalence and reduction of agents extend naturally those of processes.

## Static equivalence $A \approx_s B$

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$A$  and  $B$  give **statically indistinguishable knowledge** to the adversary.

Intuitively,  $A \approx_s B$  when

- **equality tests** cannot differentiate  $A$  from  $B$ :  
 $M = N$  with the values of the variables given by  $A \Leftrightarrow$   
 $M = N$  with the values of the variables given by  $B$ ;
- $M$  is **equal to a name** with the values of the variables given by  $A \Leftrightarrow$  it is with the values of the variables given by  $B$ .

The adversary can detect whether a term is a name by using it as a channel.

## Labeled reduction

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**Labeled reduction** represents the reduction of a process in interaction with a context.

$$a^\eta(\lambda x).P \xrightarrow{a^\eta(\lambda M)} P\{M/x\} \quad (M \text{ and } \eta \text{ closed}) \quad (\text{In})$$

$$\bar{a}^\eta(M).P \xrightarrow{\bar{a}^\eta(x)} \{M/x\}, P \quad (\eta \text{ closed}) \quad (\text{Out})$$

$$\frac{P \xrightarrow{\bar{a}^\uparrow(x)} A}{c[P] \xrightarrow{c[\bar{a}^\uparrow(x)]} c[] \circ A} \quad (\text{Loc Out})$$

$$\text{with } c[] \circ (\nu \vec{a})(\sigma, P) = (\nu \vec{a})(\sigma, c[P]) \text{ if } c \notin \vec{a}$$

Similar rule for input inside a location, etc.

## Labeled bisimilarity

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Labeled bisimilarity ( $\approx_l$ ) is the largest symmetric relation  $\mathcal{R}$  on closed agents such that  $A \mathcal{R} B$  implies:

- $A \approx_s B$ ;
- if  $A \rightarrow A'$  then there exists  $B'$  such that  $B \rightarrow^* B'$  and  $A' \mathcal{R} B'$ ;
- if  $A \xrightarrow{\alpha} A'$ , then there exists  $B'$  such that  $B \rightarrow^* \xrightarrow{\alpha} \rightarrow^* B'$  and  $A' \mathcal{R} B'$ .



## Comparison between observational equivalence and labeled bisimilarity

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**Theorem 1** *Labeled bisimilarity equals observational equivalence:  
 $P \approx_l Q$  if and only if  $P \approx Q$ .*

This result can be used to prove security properties of the previous examples.

## Conclusion

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The calculus can:

- represent **generic cryptographic primitives**
- represent **communication of code** over a network, and **manipulation of code** by cryptographic operations.
- represent **location-based security** (firewalls, sandboxes, . . .)