

# Mechanized Computational Proof of the TLS 1.3 Standard Candidate

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# Summary of the result

- **Mechanized** verification of **TLS 1.3 Draft-18** in the **computational** model.
  - + Handshake with PSK and/or DHE.
  - + Handshake with and without client authentication.
  - + 0-RTT and 0.5-RTT data, key updates.
  - - No post-handshake authentication.
  - - No version or ciphersuite negotiation: only strong algorithms.
  - - For PSK-DHE, we do not prove forward secrecy wrt. the compromise of PSK.
- We prove security properties of the initial handshake, the handshake with pre-shared key, and the record protocol using CryptoVerif.
- We compose these pieces manually.

# CryptoVerif, <http://cryptoverif.inria.fr/>

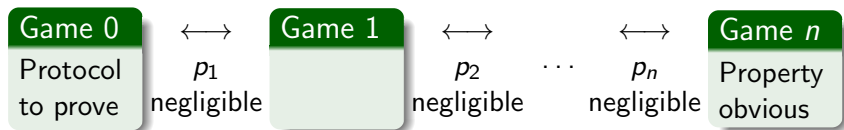
CryptoVerif is a **semi-automatic prover** that:

- works in the **computational model**.
- generates **proofs by sequences of games**.
- proves **secrecy** and **correspondence** properties.
- provides a **generic** method for specifying properties of **cryptographic primitives** which handles MACs (message authentication codes), symmetric encryption, public-key encryption, signatures, hash functions, Diffie-Hellman key agreements, ...
- works for  **$N$  sessions** (polynomial in the security parameter), with an **active adversary**.
- gives a bound on the **probability** of an attack (exact security).

# Proofs by sequences of games

CryptoVerif produces **proofs by sequences of games**, like those of cryptographers [Shoup, Bellare&Rogaway]:

- The first game is the **real protocol**.
- One goes from one game to the next by syntactic transformations or by applying the definition of security of a cryptographic primitive. The difference of probability between consecutive games is negligible.
- The last game is **“ideal”**: the security property is obvious from the form of the game.  
(The advantage of the adversary is 0 for this game.)



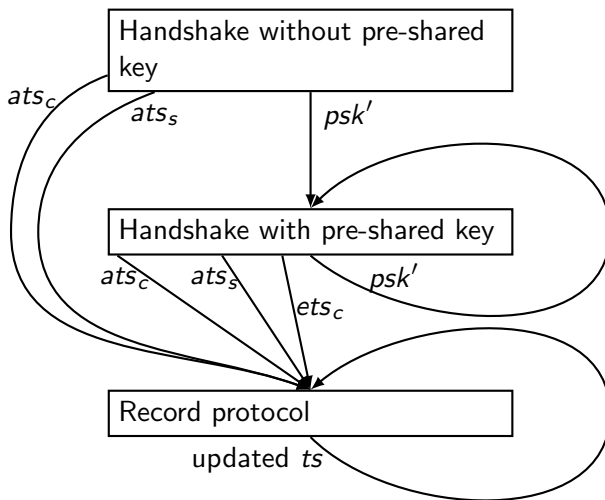
# Input and output of the tool

- 1 Prepare the input file containing
  - the specification of the **protocol** to study (initial game),
  - the **security assumptions** on the cryptographic primitives,
  - the **security properties** to prove.
- 2 Run CryptoVerif
  - Automatic proof strategy or manual guidance.
- 3 CryptoVerif outputs
  - the **sequence of games** that leads to the proof,
  - a **succinct explanation** of the transformations performed between games,
  - an upper bound of the **probability** of success of an attack.

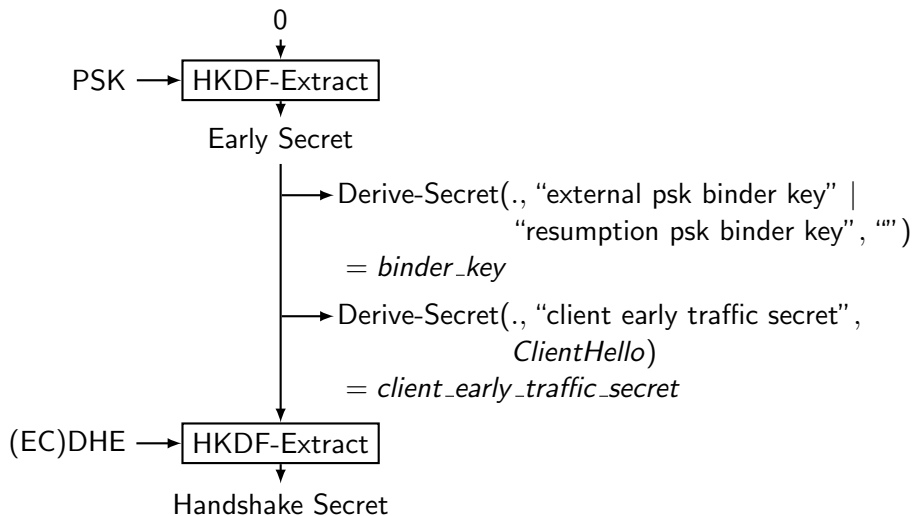
# Structure of the proof

- ① Computational assumptions
- ② Lemmas on primitives
- ③ Protocol pieces
  - Handshake without pre-shared key
  - Handshake with pre-shared key (PSK and PSK-DHE)
  - Record protocol
- ④ Compose the pieces together

# Structure of the proof: final composition



# Key schedule (Draft-18, excerpt)





# Assumptions (1)

- **Diffie-Hellman:**
  - gap Diffie-Hellman (GDH)
    - needed in particular for 0.5-RTT
  - Diffie-Hellman group of prime order
  - Diffie-Hellman group elements different from  $0^{len_H()}$ 
    - avoids confusion between handshakes with and without Diffie-Hellman exchange.
  - Diffie-Hellman group elements different from  $len_H() || \text{"TLS 1.3,"} || I || h || 0x01$ .
    - avoids collision between  $\text{HKDF-Extract}(es, e)$  and  $\text{Derive-Secret}(es, pbk, \text{""})$  or  $\text{Derive-Secret}(es, ets_c, \log_1)$ .
    - independently discovered and discussed on the TLS mailing list.
    - change in Draft-19 makes this assumption unnecessary: add a Derive-Secret stage before HKDF-Extract.

## Assumptions (2)

- **Signatures:** sign is UF-CMA.
- **Hash functions:** H is collision-resistant.
- **HMAC:**
  - $x \mapsto \text{HMAC-H}^{0^{\text{len}_H}}(x)$  and  $x \mapsto \text{HMAC-H}^{\text{kdf}_0}(x)$  are independent random oracles.
  - HMAC-H is a PRF, for keys different from  $0^{\text{len}_H}$  and  $\text{kdf}_0$ .
- **Authenticated Encryption:** IND-CPA and INT-CTXT provided the same nonce is never used twice with the same key.

# Lemmas on primitives: MAC and signatures

- $\text{mac}_H^k(m) = \text{mac}^k(H(m))$  is an SUF-CMA MAC.
- $\text{sign}_H^{sk}(m) = \text{sign}^{sk}(H(m))$  is an UF-CMA signature.

# Lemmas on primitives: key schedule

## Lemma

*When  $es$  is a fresh random value,*

- $e \mapsto \text{HKDF-Extract}(es, e)$  *and*
- $\log_1 \mapsto \text{Derive-Secret}(es, \text{ets}_c, \log_1)$   
*are indistinguishable from independent random functions, and*
- $k^b = \text{Derive-Secret}(es, \text{pbk}, \text{""})$  *and*
- $\text{HKDF-Extract}(es, 0^{\text{len}_H()})$   
*are indistinguishable from independent fresh random values independent from these random functions.*

- Proved using CryptoVerif.
- Similar lemmas for other parts of the key schedule.
- Used as assumption in the proof of the protocol.

# Handshake without pre-shared key: model

- Model a honest client and a honest server.
- May interact with dishonest clients and servers included in the adversary.
- Ignore negotiation (`RetryRequest`).
- Give the handshake keys to adversary:
  - The adversary can encrypt and decrypt messages.
  - The security proof does not rely on that.
- Server always authenticated.
- With and without client authentication.
- The honest client and server may be dynamically compromised.

# Handshake without pre-shared key: honest sessions

- The **client** is in a **honest session** if
  - the server public key is the one of the honest server, and
  - the honest server is not compromised, or it is compromised and the messages received by the client have been sent by the honest server.
- The **server** is in a **honest session** if
  - client authenticated:
    - the client public key is the one of honest client, and
    - the honest client is not compromised, or it is compromised and the messages received by the server have been sent by the honest client.
  - client not authenticated: the Diffie-Hellman share received by the server has been sent by the honest client.

# Handshake without pre-shared key: security (1)

- **Key authentication:**
  - If the honest client terminates a honest session, then the honest server has accepted a session with that client, and they agree on:
    - keys  $ats_c$ ,  $ats_s$ , and  $ems$ ,
    - all messages until the server `Finished` message.
  - If the honest server terminates a honest session, then the honest client has accepted a session with that server, and they agree on the keys and on all messages.
- **Replay prevention:** the previous properties are injective.
- **Key secrecy:** the keys
  - $ats_c$ ,  $ems$ ,  $psk'$  client side, when the client terminates a honest session;
  - $ats_s$  server side, when the server sends its `Finished` message and the received Diffie-Hellman share comes from the client (for 0.5-RTT)are indistinguishable from independent fresh random values.

# Handshake without pre-shared key: security (2)

- **Same key:**
  - If the honest client terminates a honest session and the honest server has accepted a session with the same messages, then they have the same key.
  - If the honest server terminates a honest session and the honest client has accepted a session with the same messages, then they have the same key.
- **Unique channel identifier:**
  - $psk'$  or  $H(\log_7)$ :  
If a client session and a server session have the same  $psk'$  or  $H(\log_7)$ , then all their parameters are equal (collision-resistance).
  - $ems$ :  
If a client session and a server session have the same  $ems$ , then they have the same  $\log_4$  (collision-resistance), so all their parameters are equal (CryptoVerif).



# Handshake without pre-shared key: guidance

- Signature under  $sk_S$ .
- Introduce tests to distinguish cases, depending on
  - whether the Diffie-Hellman share received by the server is a share  $g^{x'}$  from the client,
  - and whether the Diffie-Hellman share received by the client is the share  $g^y$  generated by the server upon receipt of  $g^{x'}$ .
- Random oracle assumption on  $x \mapsto \text{HMAC-H}^{\text{kdfo}}(x)$ .
- Replace variables that contain  $g^{x'y}$  with their values to make equality tests  $m = g^{x'y}$  appear.
- Gap Diffie-Hellman assumption.
- $\Rightarrow$  the handshake secret  $hs$  is a fresh random value.
- Lemmas on key schedule  $\Rightarrow$  other keys are fresh random values.
- MAC.
- Signature under  $sk_C$ .

# Handshake with pre-shared key: model

- Includes handshakes with and without Diffie-Hellman exchange.
- Includes 0-RTT.
- Ignore the ticket  $\text{enc}^{k_t}(psk)$ ; consider a honest client and a honest server that share the PSK.
- Give the handshake keys to adversary (as before).
- Certificates optional, since the client and server are already authenticated by the PSK.

# Handshake with pre-shared key: security (1)

Same properties as for the initial handshake, but

- **No compromise of PSK.**
  - Limitation of CryptoVerif: cannot prove forward secrecy wrt. to the compromise of PSK for PSK-DHE.
- Weaker properties for 0-RTT:
  - **Key authentication:** No authentication for  $ets_c$ :
    - several binders, and only one of them is checked;
    - the adversary can alter the others, yielding a different  $ets_c$  server-side.
  - **Replay prevention:** No replay protection for  $ets_c$ .
  - **Secrecy of keys:** The keys  $ets_c$  server-side are not independent of each other, due to the replay.

## Handshake with pre-shared key: security (2)

For 0-RTT, we show:

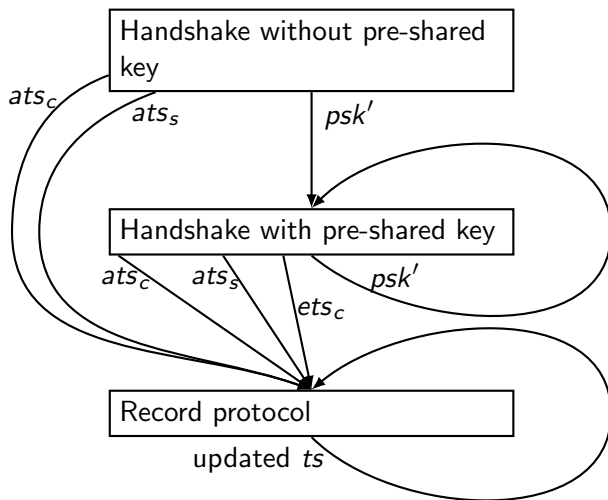
- **Client-side:** The keys  $ets_c$  are indistinguishable from independent random values.
- **Server-side:**
  - If the received ClientHello message has been sent by the client, then this session matches a session of the client with same key  $ets_c$ .
  - Otherwise,
    - If the ClientHello message has been received before, then the key  $ets_c$  computed by the server is the same as in the previous session with the same ClientHello message.
    - Otherwise, the key  $ets_c$  computed by the server is indistinguishable from a fresh random value, independent from other keys.

# Record protocol

The client and the server share a fresh random traffic secret.

- **Key secrecy:** The updated traffic secret is indistinguishable from a fresh random value.
- **Message secrecy:** When the adversary provides two sets of plaintexts  $m_i$  and  $m'_i$  of the same padded length, it is unable to determine which set is encrypted, even when the updated traffic secret is leaked.
- **Message Authentication:** If a message  $m$  is decrypted by the receiver with a counter  $c$ , then the message  $m$  has been encrypted and sent by an honest sender with the same counter  $c$ .
- **Replay Prevention:** The authentication property above is injective.

# Composition



# Composition: main theorem (informal)

- System  $S$ : key exchange;  $A$  and  $B$  obtain a key such that:
  - **Key secrecy**: The keys obtained by  $A$  are indistinguishable from independent random values.
  - **One-way injective authentication**: For each session of  $B$  that obtains a key  $k$  after sending/receiving  $\widetilde{msg}$ , there is a distinct session of  $A$  that obtains the key  $k$  after sending/receiving  $\widetilde{msg}$ .
  - **Same key**: If  $B$  obtains a key  $k$  after sending/receiving  $\widetilde{msg}$  and  $A$  obtains a key  $k'$  after sending/receiving  $\widetilde{msg}$ , then  $k = k'$ .
- System  $S'$  assumes a fresh random key shared by  $A'$  and  $B'$ .
- The composed system  $S_{composed}$  runs the key exchange followed by  $A'$  with the key obtained by  $A$  and  $B'$  with the key obtained by  $B$ .
- We have:
  - $S_{composed}$  is indistinguishable from an adversary using  $S$  and
  - $S_{composed}$  is indistinguishable from an adversary using  $S'$

The security properties of  $S$  and  $S'$  carry over to  $S_{composed}$ .

# Composition

- The previous theorem allows to perform most compositions.
- More tricky composition theorems for **0-RTT**, because the properties are weaker.
- A simpler composition theorem for **key update**.



# Conclusion

- **Mechanized** verification of **TLS 1.3 Draft-18** in the **computational** model.
  - + Handshake with PSK and/or DHE.
  - + Handshake with and without client authentication.
  - + 0-RTT and 0.5-RTT data, key updates.
  - - No post-handshake authentication.
  - - No version or ciphersuite negotiation: only strong algorithms.
  - - For PSK-DHE, we do not prove forward secrecy wrt. the compromise of PSK.
- **CryptoVerif** proves properties of the handshake with (resp. without) pre-shared-key and of the record protocol.
- We infer properties of the whole system by **manual composition**.
- **Modular** approach essential to be able to handle such a complex protocol.
- TLS 1.3 Draft-18 is **well-designed** to allow such a proof.