A Coq Framework For Verified Property-Based Testing

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(part of QuickChick)
Coq Verification Is Expensive

• When designing and verifying real systems, *most enlightenment comes from counterexamples*
• *but finding counterexamples via failed proofs very costly*
• Want to *find counterexamples as early as possible*
• *Counterexample convinces engineer better than failed proof*
• *Designs evolve*, definitions and properties often wrong
• Even when design correct & stable, proving still costly: countless iterations for *discovering lemmas and invariants*
• *this is the itch we’re trying to scratch with QuickChick*
**QuickChick**: Property-Based Testing for Coq

- We believe that property-based testing can
  - lower the cost of Coq proofs
  - become a part of the Coq proving process
    (similarly to Isabelle, ACL2, PVS, TLA+, etc)

- Not there yet ... but at the moment we have
  - a working clone of Haskell’s QuickCheck
    - Prototype Coq plugin written mostly in Coq itself
      [https://github.com/QuickChick](https://github.com/QuickChick)
    - various other prototypes and experiments
    - lots of ideas we’re trying out
Collaborators

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• writing our testing framework in Coq enables proving formal statements about testing itself
• this is the main topic of this talk
Verified Property-Based Testing? Why?

1. QuickChick is not push button
   - users will always have to write some code
     - property checkers (efficiently executable variants of properties)
     - property-based generators (producing data satisfying properties)
   - writing correct probabilistic programs is hard
   - easy to test things badly and not notice it until proving (e.g. test weaker property); this reduces benefit of testing
   - when testing finds no bugs, how can we know that we are testing things right? are we even testing the right thing?
     - answer #1: formal verification
     - answer #2: polarized mutation testing
Verified Property-Based Testing? Why?

2. Need to trust QuickChick itself
   – Subtle bugs found in Haskell QuickCheck even after 14 years of widespread usage
   – The more smarts we add to QuickChick, the bigger this issue becomes
   – Any extension we make needs to be correct
     • e.g. we would like to work out the metatheory of our upcoming property-based generator language
     • but for this we need at first define what generator and checker correctness means
A Coq Framework for Verified PBT

- Formally verify QuickChick generators and checkers
  - wrt high-level properties they are supposed to test
- Methodology for verification of probabilistic programs
  - abstraction: reasoning about the sets of outcomes a they can produce with non-zero probability
- Framework integrated in QuickChick, used to verify
  - almost all the QuickChick combinators
  - red-black trees and noninterference examples
- Modular, scalable, requires minimal code changes
A QUICK INTRODUCTION TO QUICKCHICK
Red-Black Trees Implementation

Inductive color := Red | Black.

Inductive tree :=
  | Leaf : tree

Fixpoint ins x s :=
  match s with
    | Leaf => Node Red Leaf x Leaf
    | Node c a y b => if x < y then balance c (ins x a) y b
      else if y < x then balance c a y (ins x b)
        else Node c a x b
  end.

Definition makeBlack t :=
  match t with
    | Leaf => Leaf
    | Node _ a x b => Node Black a x b
  end.

Definition insert x s := makeBlack (ins x s).
Declarative Proposition

(* Red-Black Tree invariant: declarative definition *)
Inductive is_redblack' : tree -> color -> nat -> Prop :=
  | IsRB_leaf: forall c, is_redblack' Leaf c 0
  | IsRB_r: forall n tl tr h,
    is_redblack' tl Red h -> is_redblack' tr Red h ->
    is_redblack' (Node Red tl n tr) Black h
  | IsRB_b: forall c n tl tr h,
    is_redblack' tl Black h -> is_redblack' tr Black h ->
    is_redblack' (Node Black tl n tr) c (S h).

Definition is_redblack t := exists h, is_redblack' t Red h.

Definition insert_preserves_redblack : Prop :=
  forall x s, is_redblack s -> is_redblack (insert x s).

(* Declarative Proposition *)
Lemma insert_preserves_redblack_correct : insert_preserves_redblack.
Abort. (* if this wasn't about testing, we would just prove this *)
Property Checker
(efficiently executable definitions)

Definition `is_black_balanced` (t : tree) : bool :=
  isSome (black_height_bool t).

Fixpoint `has_no_red_red` (t : tree) : bool :=
  match t with
    | Leaf => true
    | Node Red (Node Red _) _ => false
    | Node Red _ (Node Red _) => false
    | Node _ tl _ tr => has_no_red_red tl && has_no_red_red tr
  end.

Definition `is_redblack_bool` (t : tree) : bool :=
  is_black_balanced t && has_no_red_red t.

Definition `insert_is_redblack_checker` : Gen QProp :=
 forall arbitrary (fun n =>
    (forall genTree (fun t =>
      (is_redblack_bool t =>
        is_redblack_bool (insert n t) : Gen QProp)) : Gen QProp)).
Definition genColor := elements Red [Red; Black].

Fixpoint genAnyTree_max_height (h : nat) : Gen tree :=
  match h with
  | 0 => returnGen Leaf
  | S h' =>
    bindGen genColor (fun c =>
      bindGen (genAnyTree_max_height h') (fun t1 =>
        bindGen (genAnyTree_max_height h') (fun t2 =>
          bindGen arbitraryNat (fun n =>
            returnGen (Node c t1 n t2)))))
  end.

Definition genAnyTree : Gen tree := sized genAnyTree_max_height.

QuickCheck testInsertNaive.

*** Gave up! Passed only 3 tests
Discarded: 200
Finding a Bug

```haskell
Fixpoint has_no_red_red (t : tree) : bool :=
match t with
| Leaf => true
| Node Red (Node Red ___ _) ___ => false
| Node Red ___ (Node Red ___ ) => false
| Node _ tl _ tr => has_no_red_red tr && has_no_red_red tr end.
```

QuickCheck testInsertNaive.

Node Black (Node Red (Node Red (Leaf) 63 (Leaf)) 155 (Node Red (Leaf) 55 (Node Red

*** Failed! After 4021 tests and 0 shrinks

```haskell
```
Generator for Red-Black Trees
(handwritten property-based generator)

Fixpoint genRBTree_height (h : nat) (c : color) :=
  match h with
  | 0 =>
    match c with
    | Red => returnGen Leaf
    | Black => oneof (returnGen Leaf)
    [returnGen Leaf;
      bindGen arbitraryNat (fun n =>
        returnGen (Node Red Leaf n Leaf))]
  end
  | S h =>
    match c with
    | Red =>
      bindGen (genRBTree_height h Black) (fun t1 =>
        bindGen (genRBTree_height h Black) (fun t2 =>
          bindGen arbitraryNat (fun n =>
            returnGen (Node Black t1 n t2))))
    | Black => ..........

Definition genRBTree := sized (fun h => genRBTree_height h Red).
Property-Based Generator at Work

Definition testInsert :=
  showDiscards (quickCheck (insert_is_redblack_checker genRBTTree)).
QuickCheck testInsert.

Success: number of successes 10000
  number of discards 0

in less than 4 seconds
Are we testing the right property?

VERIFIED PROPERTY-BASED TESTING

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Proving correctness of generators

Definition genColor := elements Red [Red; Black].

Lemma semElements :
  forall {A} (l: list A) (def : A),
  (semGen (elements def l)) \iff
  (fun e => List.In e l \/ (l = nil \/ e = def)).

Lemma genColor_correct:
  semGen genColor \iff (fun _ => True).
Proof.
  rewrite /genColor. intros c. rewrite semElements.
  split => // _. left.
  destruct c; by [ constructor | constructor(constructor)].
Qed.

Lemma genRBTTree_height_correct: forall c h,
  (genRBTTree_height h c) \iff (fun t => is_redblack' t c h).

Theorem genRBTTree_correct:
  semGen genRBTTree \iff is_redblack.
Proving correctness of checkers

Lemma is_redblackP :
  forall (t : tree),
      reflect (is_redblack t) (is_redblack_bool t).

Lemma semImplication:
  forall {prop : Type} {H : Checkable prop}
       (p : prop) (b : bool) (s : nat),
         semCheckerSize (b ==> p) s <-> b = true -> semCheckableSize p s.

Lemma semForAll :
  forall {A prop : Type} {H : Checkable prop} `{Show A}
       (gen : G A) (f : A -> prop) (size: nat),
         semCheckerSize (forall gen f) size <->
         forall (a : A), semSize gen size a -> semCheckableSize (f a) size.

Lemma insert_is_redblack_checker_correct:
  semChecker (insert_is_redblack_checker genRBTree) <-> insert_preserves_redblack.
Set of outcomes semantics

– semantics of a generator is a set
  • intuitively containing the values that can be generated with $>0$ probability

– semantics of a checker is a Coq proposition
Formally we define

\[
\text{Definition Ensemble (A : Type) := A -> Prop.}
\]

\[
\text{Definition set_eq \{A\} (m1 m2 : Ensemble A) :=}
\]
\[
\quad \text{forall (a : A), m1 a <-> m2 a.}
\]
\[
\text{Infix "<-->" := set_eq (at level 70, no associativity) : sem_gen_scope.}
\]

\[
\text{Definition semSize \{A : Type\} (g : Gen A) (size : nat) : Ensemble A :=}
\]
\[
\quad \text{fun a => exists seed, (unGen g) seed size = a.}
\]

\[
\text{Definition semGen \{A : Type\} (g : Gen A) : Ensemble A :=}
\]
\[
\quad \text{fun a => exists size, semSize g size a.}
\]

\[
\text{Record QProp : Type := MkProp \{ unProp : Rose Result \}.}
\]

\[
\text{Definition Checker : Type := Gen QProp.}
\]

\[
\text{Definition semChecker (P : Checker) : Prop :=}
\]
\[
\quad \text{forall s qp, semSize P s qp -> success qp = true.}
\]
QuickChick/Proof Organization

User Code → High-Level Generators & Checkers

↓

Low-Level Generators

↓

Internal Primitives (OCaml)
Internal Primitives & Axiom(s)

• random seed type + 8 primitive functions written only in OCaml and only assumed in Coq
• 5 axioms about these primitive functions
  – 4 of them would disappear if we implemented a splittable random number generator in Coq
  – remaining axiom is inherent to our abstraction!

\[
\text{Axiom } \text{rndSplitAssumption :} \\
\forall s1 \ s2 : \text{RandomSeed}, \exists s, \text{rndSplit } s = (s1,s2).
\]

• makes the type RandomSeed infinite in Coq, while in OCaml it is finite (seeds are bounded integers)
• we assume real randomness (an oracle) in the proofs, but can only implement pseudo-randomness
Lemmas for Low-Level QC Generators (10)

- they rely on primitives and concrete representation of Gen

Lemma \( \text{semReturnSize} : \forall A \ (x : A) \ (\text{size} : \text{nat}), \) 
\[ \text{semSize} \ (\text{returnGen} \ x \ \text{size}) \ \text{size} \iff \text{eq} \ x. \]

Lemma \( \text{semBindSize} : \forall A \ B \ (g : G A) \ (f : A \to G B) \ (\text{size} : \text{nat}), \) 
\[ \text{semSize} \ (\text{bindGen} \ g \ f) \ \text{size} \iff (\text{fun} \ b \ => \ \exists a, \ (\text{semSize} \ g \ \text{size}) \ a \ \land \ (\text{semSize} \ (f \ a) \ \text{size}) \ b). \]

- bind proof crucially relies on axiom about \( \text{rndSplit} \)
- we can’t abstract over the sizes (existentially quantify)

Lemma \( \text{semSizedSize} : \)
\[ \forall A \ (f : \text{nat} \to G A), \]
\[ \text{semGen} \ (\text{sized} \ f) \iff (\text{fun} \ a \ => \ \exists n, \ \text{semSize} \ (f \ n) \ n \ a). \]

Lemma \( \text{semResize} : \)
\[ \forall A \ (n : \text{nat}) \ (g : G A), \ \text{semGen} \ (\text{resize} \ n \ g) \iff \text{semSize} \ g \ n. \]
High-Level Generators & Checkers (12)

Lemma semElements
  forall {A} (l: list A) (def : A),
  (semGen (elements def l)) <->
  (fun e => List.In e l \/
   (l = nil \/
    e = def)).

Lemma semFrequency: forall {A} (l : list (nat * G A)) (def : G A),
  semGen (frequency def l) <->
  (fun e => (exists n, exists g, (List.In (n, g) l \/
    semGen g e \/
    n <> 0)) \/
    ((l = nil \/
      (forall x, List.In x l -> fst x = 0)) \/
     semGen def e)).

Lemma semImplication:
  forall {prop : Type} {H : Checkable prop}
  (p : prop) (b : bool) (s : nat),
  semCheckerSize (b => p) s <-> b = true -> semCheckableSize p s.

Lemma semForAll :
  forall {A prop : Type} {H : Checkable prop} `{Show A}
  (gen : G A) (f : A -> prop) (size: nat),
  semCheckerSize (forall gen f) size <->
  forall (a : A), semSize gen size a -> semCheckableSize (f a) size.
Summary

• Coq framework for verified PBT
• Integrated in QuickChick
  – https://github.com/QuickChick
• Reasoning about sets of outcomes
• The first verified QuickCheck implementation
• Examples: red-black trees and noninterference
• Modular, scalable, minimal code changes
Future Work

• More proof automation and infrastructure
  – changing to efficient data representations
  – SMT-based verification for sets of outcomes
• Verify property-based generator language
• Probabilistic verification
• Splittable RNG in Coq
• Try to reduce testing cost, now significant
  – break even point very much problem-specific
THANK YOU

Code at https://github.com/QuickChick