Composition of Password-based Protocols

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Other protocols may be executed in parallel

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Cryptographic process calculi and composition

Cryptographic pi calculi, e.g., the applied pi calculus or the spi calculus are well-suited for reasoning about composition

if $P_1$ is secure and $P_2$ is secure then $P_1 \mid P_2$ is secure

There are two main reasons for this

1. processes are shown secure in the presence of an arbitrary environment
2. processes do not share any secrets (this is due to the scope operator)

One would like to show that

if $\nu s. P_1$ is secure and $\nu s. P_2$ is secure then $\nu s. (P_1 \mid P_2)$ is secure

which does not hold in general

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Cryptographic process calculi and composition

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if $P_1 \simeq S_1$ and $P_2 \simeq S_2$ then $P_1 \parallel P_2 \simeq S_1 \parallel S_2$

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Guessing attacks

Solution: do not share secrets between protocols, but this is not always possible

Passwords: it is not realistic that users never re-use the same password

In this talk we investigate the question:

\[ \nu.p.P_1 \text{ and } \nu.p.P_2 \text{ are resistant against guessing attacks on } p \]
\[ \nu.p.(P_1 \mid P_2) \text{ also resistant against guessing attacks on } p? \]

guessing or dictionary attacks consists of two phases

1. the attacker one or several sessions of a protocol
2. the attacker tries offline each of the possible passwords (out of a dictionary) on the data collected during the first phase
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Passive guessing or dictionary attacks consists of two phases

1 the attacker eavesdrops one or several sessions of a protocol

2 the attacker tries offline each of the possible passwords (out of a dictionary) on the data collected during the first phase
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\]

Active guessing or dictionary attacks consists of two phases

1. the attacker interacts with one or several sessions of a protocol
2. the attacker tries offline each of the possible passwords (out of a dictionary) on the data collected during the first phase
Terms and equational theories

We consider a simple process language inspired by the applied pi calculus to describe protocols

Messages are modeled using terms

- Abstract algebra given by a signature, i.e. a set of function symbols with arities
- Equivalence relation ($=_{E}$) on terms induced by an equational theory

Example (equational theory)

Consider the signature $\Sigma_{\text{enc}} = \{\text{sdec, senc, adec, aenc, pk, } \langle \rangle, \text{proj}_1, \text{proj}_2\}$

\[
\begin{align*}
\text{sdec(senc}(x, y), y) &= x \\
\text{senc(}\text{sdec}(x, y), y) &= x \\
\text{proj}_i(\langle x_1, x_2 \rangle) &= x_i \quad (i \in \{1, 2\}) \\
\text{adec(aenc}(x, \text{pk}(y)), y) &= x
\end{align*}
\]
Frames and deduction

Terms are regrouped into frames: a set of secrets + a substitution

\[ \nu \tilde{n}.(\{M_1/x_1\} \mid \ldots \mid \{M_n/x_n\}) \]

Definition (Deduction)

\[ \nu \tilde{n}.\sigma \vdash \text{E} \ M \ \text{iff there exists} \ N \ \text{such that} \ fn(N) \cap \tilde{n} = \emptyset \ \text{and} \ N\sigma = \text{E} \ M. \]

We call \( N \) a recipe of the term \( M \).

Example

Let \( \phi = \nu k, s_1.\{\text{senc}(\langle s_1, s_2 \rangle, k)/x_1, k/x_2\} \)

<table>
<thead>
<tr>
<th>( \phi \vdash <em>{\text{E}</em>{\text{enc}}} k )</th>
<th>( \phi \vdash <em>{\text{E}</em>{\text{enc}}} s_1 )</th>
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<tr>
<td>( x_2 )</td>
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Static equivalence

**Definition (Static equivalence)**

\( M \) and \( N \) are equal in \( \phi \), written \((M =_E N)\phi\), if \( \phi =_\alpha \nu \tilde{n}.\sigma \), \( M\sigma =_E N\sigma \), and \( \tilde{n} \cap (\text{fn}(M) \cup \text{fn}(N)) = \emptyset \).

\( \phi_1 \) and \( \phi_2 \) are statically equivalent, \( \phi_1 \approx_E \phi_2 \), when:

- \( \text{dom}(\phi_1) = \text{dom}(\phi_2) \), and
- for all terms \( M, N \), \((M =_E N)\phi_1 \) iff \((M =_E N)\phi_2 \)

**Example**

\[
\phi = \nu k.\{\text{senc}(s_0,k)/x_1, k/x_2\} \not\approx \nu k.\{\text{senc}(s_1,k)/x_1, k/x_2\} = \phi'
\]

because of the test \((\text{sdec}(x_1,x_2), s_0)\)

However,

\[
\nu k.\{\text{senc}(s_0,k)/x_1\} \approx \nu k.\{\text{senc}(s_1,k)/x_1\}
\]
Syntax of the process language

Plain processes

\[ P, Q, R := \]

\[ 0 \quad \text{null process} \]

\[ P \mid Q \quad \text{parallel composition} \]

\[ \text{in}(x).P \quad \text{message input} \]

\[ \text{out}(M).P \quad \text{message output} \]

\[ \text{if } M = N \text{ then } P \text{ else } Q \quad \text{conditional} \]

Extended processes \( A, B, C := P \mid A \mid B \mid \nu n.A \mid \{^M/x\} \)

Frame of a process: \( \phi(A) \) obtained by replacing plain processes by 0

Example

\[
A = \nu s, k_1. (\text{in}(x). \text{if sdec}(x, k_1) = s \text{ then out}(a) \mid \{^\text{senc}(s, k_1)/x\} \mid \\
\quad \nu k_2. \text{out}(\text{senc}(s, k_2)))
\]

\[
\phi(A) = \nu s, k_1. (0 \mid \{^\text{senc}(s, k_1)/x\} \mid \nu k_2. 0)
\]
Semantics of the process language

Structural equivalence: the smallest equivalence relation closed by application of evaluation contexts and such that

| Par-0 | $A | 0 \equiv A$ | New-Par | $A | \nu n.B \equiv \nu n.(A | B)$ |
| Par-C | $A | B \equiv B | A$ |
| Par-A | $(A | B) | C \equiv A | (B | C)$ |
| New-C | $\nu n_1.\nu n_2.A \equiv \nu n_2.\nu n_1.A$ |

Operational semantics: smallest relation between extended processes which is closed under structural equivalence ($\equiv$) and such that

In

$$\text{in}(x).P \xrightarrow{\text{in}(M)} P\{^M/x\}$$

Out

$$\text{out}(M).P \xrightarrow{\text{out}(M)} P \mid \{^M/x\} \quad \text{where } x \text{ is a fresh variable}$$

Then

$$\text{if } M = N \text{ then } P \text{ else } Q \xrightarrow{\tau} P \quad \text{where } M \equiv_E N$$

Else

$$\text{if } M = N \text{ then } P \text{ else } Q \xrightarrow{\tau} Q \quad \text{where } M \not\equiv_E N$$

Cont.

$$A \xrightarrow{\ell} B$$

$$C[A] \xrightarrow{\ell} C[B] \quad \text{where } C \text{ is an evaluation context}$$

If $\ell = \text{in}(M)$ then $\phi(C[A]) \vdash_E M$
Semantics of the process language

**Structural equivalence:** the smallest equivalence relation closed by application of evaluation contexts and such that

| Par-0 | $A | 0 \equiv A$ | New-Par | $A | \nu n.B \equiv \nu n.(A | B)$ |
|-------|----------------|---------|----------------------------------|
| Par-C | $A | B \equiv B | A$ |         |                                  |
| Par-A | $(A | B) | C \equiv A | (B | C)$ | New-C  | $\nu n_1.\nu n_2.A \equiv \nu n_2.\nu n_1.A$ |

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if $\ell = \text{in}(M)$ then $\phi(C[A]) \vdash_E M$
Definition of guessing attacks

Definition from [Baudet05] (inspired from [Corin et al.03])

Definition (Passive guessing attacks)

\[ \nu w. \phi \text{ is resistant to guessing attacks against } w \text{ iff } \]

\[ \nu w.(\phi | \{w/x\}) \approx \nu w.(\phi | \nu w'.\{w'/x\}) \]

Definition (Active guessing attacks)

A is resistant to guessing attack against w if, for every process B such that A \rightarrow^* B, we have that \( \phi(B) \) is resistant to guessing attacks against w.
Definition of guessing attacks

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**Definition (Passive guessing attacks)**

\( \nu w. \phi \) is **resistant to guessing attacks** against \( w \) iff

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**Definition (Active guessing attacks)**

\( A \) is **resistant to guessing attack** against \( w \) if, for every process \( B \) such that \( A \rightarrow^* B \), we have that \( \phi(B) \) is resistant to guessing attacks against \( w \).
Composing resistance against passive guessing attacks

**Proposition**

The three following statements are equivalent:

1. \( \nu w.\phi \mid \{w/x\} \approx \nu w.\phi \mid \nu w'.\{w'/x\} \)

2. \( \phi \approx \nu w.\phi \)

3. \( \phi \approx \phi\{w'/w\} \)

[Corin et al.03]

[Baudet05]

It follows from the last point that passive guessing attacks do compose!

**Corollary**

If \( \nu w.\phi_1 \) and \( \nu w.\phi_2 \) are resistant to guessing attacks against \( w \), then \( \nu w.(\phi_1 \mid \phi_2) \) is also resistant to guessing attacks against \( w \).
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If $\nu w.\phi_1$ and $\nu w.\phi_2$ are resistant to guessing attacks against $w$ then $\nu w.(\phi_1 \mid \phi_2)$ is also resistant to guessing attacks against $w$. 

[Baudet 05]
Unbounded number of sessions for free

A consequence for password-only protocols:
if one session of the protocol is safe against a passive adversary then an unbounded number of sessions are safe against a passive adversary

Example (EKE protocol [BellovinMerritt92])

A → B : senc(pk(k)), w) (EKE.1)
B → A senc(aenc(r, pk(k)), w) (EKE.2)
A → B senc(na, r) (EKE.3)
B → A senc(⟨na, nb⟩, r) (EKE.4)
A → B senc(nb, r) (EKE.5)

\[ \phi = \nu k, r, na, nb. \{ senc(pk(k), w) / x_1, senc(aenc(r, pk(k)), w) / x_2, senc(na, r) / x_3, senc(⟨na, nb⟩, r) / x_4, senc(nb, r) / x_5 \} \]

We indeed have that \( \nu w.(\phi \mid \{ w / x \}) \approx \nu w'.(\phi \mid \{ w' / x \}) \)
Resistance against active guessing attacks does not compose in general!

Counter-example

Suppose the following equational theory

\[
f_3(f_2(f_1(x, y), x, z), y) = x
\]

\[
A_1 = \nu n_1\cdot\text{out}(f_1(w, n_1))\cdot\text{in}(x)\cdot\text{out}(f_3(x, n_1))
\]

\[
A_2 = \nu n_2\cdot\text{in}(y)\cdot\text{out}(f_2(y, w, n_2))
\]

\(\nu w.A_1\) and \(\nu w.A_2\) resist against guessing attacks

A guessing attack on \(\nu w.(A_1 \mid A_2)\):

\[
\nu w.(A_1 \mid A_2) \rightarrow^{*} \nu w, n_1, n_2.(\{f_1(w, n_1)/x_1\} \mid \{f_2(f_1(w, n_1), w, n_2)/x_2\} \mid \{w/x_3\})
\]

\(\nu w.\phi \mid \{w/x\} \not\approx \nu w.\phi \mid \nu w'.\{w'/x\}\)
Well-taged protocols and composition

Intuitively, a protocol is well-tagged w.r.t. a secret \( w \) if all the occurrences of \( w \) are of the form \( h(\alpha, w) \)

**Definition (well-tagged)**

\( M \) is \( \alpha \)-tagged w.r.t. \( w \) if there exists \( M' \) s.t. \( M'\{h(\alpha, w)/w\} \equiv E M \). A term is said well-tagged w.r.t. \( w \) if it is \( \alpha \)-tagged for some name \( \alpha \).

\( A \) is \( \alpha \)-tagged if any term occurring in it is \( \alpha \)-tagged. An extended process is well-tagged if it is \( \alpha \)-tagged for some name \( \alpha \).

Well-tagged processes compose!

**Theorem (composition result)**

Let \( A_1 \) be \( \alpha \)-tagged and \( A_2 \) be \( \beta \)-tagged w.r.t. \( w \).

If \( \nu w. A_1 \) and \( \nu w. A_2 \) are resistant to guessing attacks against \( w \) then \( \nu w. (A_1 \mid A_2) \) is also resistant to guessing attacks against \( w \).
Well-tagged protocols and composition

Intuitively, a protocol is well-tagged w.r.t. a secret $w$ if all the occurrences of $w$ are of the form $h(\alpha, w)$

**Definition (well-tagged)**

$M$ is $\alpha$-tagged w.r.t. $w$ if there exists $M'$ s.t. $M'\{h(\alpha, w)/w\} =_E M$. A term is said well-tagged w.r.t. $w$ if it is $\alpha$-tagged for some name $\alpha$.

$A$ is $\alpha$-tagged if any term occurring in it is $\alpha$-tagged. An extended process is well-tagged if it is $\alpha$-tagged for some name $\alpha$.

Well-tagged processes compose!

**Theorem (composition result)**

Let $A_1$ be $\alpha$-tagged and $A_2$ be $\beta$-tagged w.r.t. $w$. If $\nu w.A_1$ and $\nu w.A_2$ are resistant to guessing attacks against $w$ then $\nu w.(A_1 \mid A_2)$ is also resistant to guessing attacks against $w$. 
A secure transformation

**Theorem**

If $\nu w. A$ is resistant to guessing attacks against $w$ then $\nu w. (A^{h(\alpha, w)/w})$ is also resistant to guessing attacks against $w$.

Easy, syntactic transformation: thumbrule for good design?

Remark on other transformations:

- replacing $w$ by $\langle w, \alpha \rangle$ does not guarantee composition
- tagging encryptions (used in [CortierDelaitreDelaune07] to ensure composition of other properties) would add guessing attacks
Conclusion and future work

Passive guessing attacks do compose

Active guessing attacks do not compose in general

but for well-taged protocols

Secure transformation to obtain well-tagged protocols

Future work

Avoid tags: are there (interesting) classes of equational theories for which guessing attacks compose?

Other forms of composition:

- composition for observational equivalence
- sequential composition