From a Concurrency Course to Automatic Verification of Process Equivalences

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Concurrence 2

Communication par canal et Pi-calcul

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Plan

1. Mémoire partagée
2. Réseaux de Petri
3. CSP, CCS, Meije, ACP
4. Pi-calcul (définitions)
5. Pi-calcul (exemples)
6. Pi-calcul polyadique
7. Pi-calcul asynchrone
Concurrence 3

Pi-calcul

Pict

Pi-calcul d'ordre supérieur

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Plan

1. Codage de l'arithmétique
2. Codage λ-calcul
3. Codage des structures de données (listes, arbres)
4. Typage des canaux
5. Pi-calcul asynchrone
6. Machines,Expériences,Equivalences
Concurrence

Bisimulations

Pict

Pi-calcul d’ordre supérieur

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Plan

1. Machines, Expériences, Equivalences
2. Pi-calcul avec abstractions
3. Pi-calcul d’ordre supérieur
2001-2002: ProVerif: automatic security protocol verifier

- Protocol: Pi calculus + cryptography
- Properties to prove: Strong secrecy

Automatic translator

- Horn clauses
- Derivability queries

Resolution with selection

- The property is true
- Potential attack

(also owes much to Martín Abadi)
ProVerif initially verified only properties on behaviors (traces) of protocols (secrecy of keys, correspondences).

Many important properties can be formalized as process equivalences, not as properties on behaviors:

- secrecy of a boolean $x$ in $P(x)$: $P(\text{true}) \approx P(\text{false})$
- the process $P$ implements an ideal specification $Q$: $P \approx Q$

Equivalences are usually proved by difficult, long manual proofs.

Much research on this topic, using in particular bisimulation techniques (e.g., Boreale et al).
Goal: extend tools designed for proving properties of behaviors (here ProVerif) to the proof of process equivalences.

- We focus on equivalences between processes that differ only by the terms they contain, e.g., $P(\text{true}) \approx P(\text{false})$.

Many interesting equivalences fall into this category.

- Biprocesses represent pairs of processes that differ only by the terms they contain.

  $P(\text{true})$ and $P(\text{false})$ are variants of a biprocess $P(\text{diff[true, false]})$.

  The variants give a different interpretation to $\text{diff[true, false]}$, $\text{true}$ for the first variant, $\text{false}$ for the second one.
We introduce a new operational semantics for biprocesses:

A biprocess reduces when both variants reduce in the same way and after reduction, they still differ only by terms (so can be written using `diff`).

We establish $P(\text{true}) \approx P(\text{false})$ by reasoning on behaviors of $P(\text{diff}[\text{true}, \text{false}])$:

If, for all reachable configurations, both variants reduce in the same way, then we have equivalence.

(extends to cryptography an idea by Pottier and Simonet)
The process calculus

Extension of the pi-calculus with function symbols for cryptographic primitives.

\[ M, N ::= \]
\[ x, y, z \]
\[ a, b, c, k, s \]
\[ f(M_1, \ldots, M_n) \]

\[ D ::= \]
\[ M \]
\[ \text{eval } h(D_1, \ldots, D_n) \]

\[ P, Q, R ::= \]
\[ M(x).P \]
\[ \overline{M}(\langle N \rangle).P \]
\[ \text{let } x = D \text{ in } P \text{ else } Q \]
\[ 0 \quad P \mid Q \quad !P \quad (\nu a)P \]
Representation of cryptographic primitives

Two possible representations:

- **When success/failure is visible**: destructors with rewrite rules
  
  constructor \( sencrypt \)
  
  destructor \( sdecrypt(sencrypt(x, y), y) \rightarrow x \)

  The *else* clause of the term evaluation is executed when no rewrite rule of some destructor applies.

- **When success/failure is not visible**: equations

  \[ sdecrypt(sencrypt(x, y), y) = x \]
  
  \[ sencrypt(sdecrypt(x, y), y) = x \]
Semantics

$D \Downarrow M$ when the term evaluation $D$ evaluates to $M$.
Uses rewrite rules of destructors and equations.

$\equiv$ transforms processes so that reduction rules can be applied.

Main reduction rules:

$$
\overline{N}(M).Q \mid N'(x).P \rightarrow Q \mid P\{M/x\}
$$
if $\Sigma \vdash N = N'$

(let $x = D$ in $P$ else $Q$) $\rightarrow$ $P\{M/x\}$
if $D \Downarrow M$

(let $x = D$ in $P$ else $Q$) $\rightarrow$ $Q$
if there is no $M$ such that $D \Downarrow M$
Two processes $P$ and $Q$ are **observationally equivalent** ($P \approx Q$) when the adversary cannot distinguish them.

A **biprocess** $P$ is a process with diff.

$\text{fst}(P) =$ the process obtained by replacing $\text{diff}[M, M']$ with $M$.

$\text{snd}(P) =$ the process obtained by replacing $\text{diff}[M, M']$ with $M'$.

$P$ satisfies observational equivalence when $\text{fst}(P) \approx \text{snd}(P)$.
Semantics of biprocesses

A biprocess reduces when both variants of the process reduce in the same way.

\[
\overline{N}(M).Q \mid N'(x).P \rightarrow Q \mid P\{M/x\} \quad \text{(Red I/O)}
\]

if \( \Sigma \vdash \text{fst}(N) = \text{fst}(N') \) and \( \Sigma \vdash \text{snd}(N) = \text{snd}(N') \)

\[
\text{let } x = D \text{ in } P \text{ else } Q \rightarrow P\{\text{diff}[M_1, M_2]/x\} \quad \text{(Red Fun 1)}
\]

if \( \text{fst}(D) \Downarrow M_1 \) and \( \text{snd}(D) \Downarrow M_2 \)

\[
\text{let } x = D \text{ in } P \text{ else } Q \rightarrow Q \quad \text{(Red Fun 2)}
\]

if there is no \( M_1 \) such that \( \text{fst}(D) \Downarrow M_1 \) and there is no \( M_2 \) such that \( \text{snd}(D) \Downarrow M_2 \)
Let $P_0$ be a closed biprocess.

If for all configurations $P$ reachable from $P_0$ (in the presence of an adversary), both variants of $P$ reduce in the same way, then $P_0$ satisfies observational equivalence.
Proof of observational equivalence using biprocesses

Let $P_0$ be a closed biprocess.

*If for all configurations $P$ reachable from $P_0$ (in the presence of an adversary), both variants of $P$ reduce in the same way, then $P_0$ satisfies observational equivalence.*

An adversary is represented by a plain evaluation context (evaluation context without diff), so:

*If, for all plain evaluation contexts $C$ and reductions $C[P_0] \rightarrow^* P$, both variants of $P$ reduce in the same way, then $P_0$ satisfies observational equivalence.*
Let $P_0$ be a closed biprocess.

Suppose that, for all plain evaluation contexts $C$ and reductions $C[P_0] \rightarrow^* P$,

1. the (Red I/O) rules apply in the same way on both variants.

   if $P \equiv C'[\overline{N}(M).Q | N'(x).R]$, then
   $\Sigma \vdash \text{fst}(N) = \text{fst}(N')$ if and only if $\Sigma \vdash \text{snd}(N) = \text{snd}(N')$.

2. the (Red Fun) rules apply in the same way on both variants.

   if $P \equiv C'[\text{let } x = D \text{ in } Q \text{ else } R]$, then
   there exists $M_1$ such that $\text{fst}(D) \Downarrow M_1$
   if and only if
   there exists $M_2$ such that $\text{snd}(D) \Downarrow M_2$.

Then $P_0$ satisfies observational equivalence.
Thanks Jean-Jacques for all that you taught me!

- **pi-calculus** ⇒ influence on the design of ProVerif
- **equivalences** ⇒ automatic proof of observational equivalences
  Application, e.g., to the proof of resistance to dictionary attacks

Implementation and papers at

http://www.proverif.ens.fr/